Interpretation of Transmissivity Estimates from Single-Well, Pumping Aquifer Tests

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Abstract

Interpretation of single-well tests with the Cooper-Jacob method remains more reasonable than most alternatives. Drawdowns from 628 simulated single-well tests were interpreted with the Cooper-Jacob straight-line method. Error and bias as a function of vertical anisotropy, partial penetration, specific yield, and interpretive technique were investigated for transmissivities that ranged from 10 to 10,000 m²/d. Cooper-Jacob transmissivity estimates in confined aquifers were affected minimally by partial penetration, vertical anisotropy, or analyst. Cooper-Jacob transmissivity estimates of simulated unconfined aquifers averaged twice known values. One percent of these estimates were more than 10 times known values. Transmissivity estimates were not improved by interpreting results with an unconfined solution. An empirical corrector reduced the bias and magnitude of error in Cooper-Jacob transmissivity estimates for unconfined aquifers. The empirical corrector is available as a Visual Basic for Applications function in a single-well analysis spreadsheet.

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Introduction

Single-well aquifer tests provide data in many situations in which aquifer pumping-test data from multiple wells would be a luxury for the ground-water analyst. The primary factor limiting the performance of multi-well aquifer tests is cost, particularly in areas with large depth to water (Belcher et al, 2001). Limiting analysis to existing wells predominately occurs in the arid West, but can occur in many areas. For example, irrigation wells are installed far apart to minimize interference. This separation causes adjacent wells to be poor observation wells.

The ground-water analyst faces many problems and challenges where hydraulic testing is limited to an isolated well. These problems include well-known and quantifiable effects such as borehole head loss during pumping, as well as much more difficult-to-quantify factors such as lateral and vertical anisotropy. Many effects could only be quantified with reasonable accuracy using multiple observation wells or flow logs (Hanson and Nishikawa, 1996). An analyst can still derive significant useful information from single-well tests.

Single-well aquifer tests frequently are analyzed with the Cooper-Jacob (1946) method because of its simplicity. Transmissivity is estimated by fitting a straight line to drawdowns on an arithmetic axis versus time on a logarithmic axis in a semilog plot. Drawdowns in confined and unconfined aquifers have been analyzed with the Cooper-Jacob method regardless of differences between field conditions and theory.

Single-well tests from partially penetrating wells in unconfined aquifers depart greatly from the Theis (1935) model. The Cooper-Jacob method is a simplification of the Theis solution so the production well should fully penetrate a confined, homogeneous, and isotropic aquifer. Unconfined aquifer tests are affected by vertical anisotropy and specific yield in addition to transmissivity and storage coefficient. These additional parameters control vertical gradients that are created by partial penetration and drainage from the water table. Likewise, leakage from adjacent confining beds also could affect the Cooper-Jacob method. Transmissivity likely will be overestimated where leakage occurs.

Transmissivity estimates from single-well tests in unconfined aquifers also are affected by discharge rate, test duration, and interpretive technique. The transition from the release of water from storage owing to the compressibility of the medium and fluid to drainage of pores is less likely to be observed during a test of relatively small discharge or short duration. Interpretation is hampered because a final drawdown limb, which has a slope predicted by the Cooper-Jacob method, is absent.

Purpose and Scope

This article documents how interpreting single-well, pumping aquifer tests with the Cooper-Jacob method affects transmissivity estimates. Drawdowns in 800 single-well tests were simulated with radial, MODFLOW models that were defined with transmissivities between 10 and 10,000 m²/d. Drawdowns from 628 of these simulated tests would maintain water levels above pump intakes and were interpreted with the Cooper-Jacob, straight-line method. Transmissivity error and bias as a function of vertical anisotropy, partial penetration, specific yield, and interpretive technique were investigated. Transmissivity estimates were improved with informed interpretation or with an empirical, corrector algorithm.

Approach

Effects of unmet assumptions on the reliability of Cooper-Jacob transmissivity estimates were investigated by interpreting 628 simulated drawdowns. The transmissivity of each aquifer was estimated with a Cooper-Jacob analysis of simulated drawdowns. Effects of unmet assumptions were quantified by comparing Cooper-Jacob transmissivity estimates to specified values, herein referred to as known transmissivity.

Hydraulic properties were limited to plausible ranges. Transmissivities ranged from 10 to 10,000 m²/d. Transmissivities of less than 10 m²/d were excluded because slug tests are more practical than pumping tests. A single specific storage of 5×10^{-6} m^{-1} was assigned to all aquifers because transmissivity estimates were insensitive to specific storage. Specific yields ranged from 0.05 to 0.3. Vertical anisotropies ranged between 0.02 and 0.2 and were assumed to represent a sedimentary system.

Pumping wells that penetrated between 10 and 100 percent of aquifers were analyzed. All partially penetrating wells were open at the top of the aquifer or water table (Figure 1). Wellbore storage and skin effects of the pumping wells were simulated which decreased maximum potential pumping rates. Simulation results were considered physically impossible and rejected where simulated water levels in the pumping well were less than 3 m above the bottom of the well. Wells in unconfined aquifers were simulated to penetrate 25 or more percent of the saturated thickness so water levels would remain above the pump intakes.

All single-well tests were simulated over 2-day periods to balance testing effectiveness and operational constraints. Transmissivity estimates from tests of longer duration are less ambiguous because drainage from the water table will be observed and drawdowns will follow a late-time response. Actual single-well tests typically range between 1 and 3 days in duration because of operational constraints.

Simulated Aquifer Tests

All single-well aquifer tests were simulated with two-dimensional MODFLOW models (McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1996). The production well in each model was simulated as a high conductivity zone where water was removed from the uppermost cell and flow was apportioned within MODFLOW. All models extended 100,000 m from the production well along a row that was discretized into 99 columns (Figure 1). Pumping wells were simulated in column 1 and aquifer material was simulated with columns 2 through 99. Column 2 was 0.02 m wide and the remaining columns were 1.15 times the width of the previous column. The 100-m thick aquifers were uniformly subdivided into 100 rows. All models had no flow lateral boundaries, initial heads of 0 m, and a single stress period of 50 time steps. The first time step was 0.1 second in duration. Each successive time step was 1.3 times greater than the previous one.

Hydraulic properties were modified consistently to simulate axisymmetric, radial flow. Simulated pumpage was withdrawn from column 1 so that distance from the left edge of column 1 to a column node would be equivalent to the radial distance from the pumping well. Hydraulic conductivity and storage of the ith column were multiplied by $2\pi r_i$ to simulate radial flow where r_i is the distance from the outer edge of the first column to the center of the ith column. Axisymmetric, radial flow has been solved with MODFLOW by using many layers and a single row (Reilly and Harbaugh, 1993; Clemo, 2002). A single MODFLOW layer is more convenient because input is defined easily, all conductances are computed within the BCF package, and output is checked quickly.

Pumping rates were simulated near the maximum rate that could be pumped practically from an aquifer of a given transmissivity. Pumping rates were greater in more transmissive aquifers (Table 1). Realistic wellbore-storage effects were simulated by increasing well diameters as pumping rates increased. A storage coefficient of 1 was assigned in row 1 of column 1 to simulate wellbore storage. Conductances between columns 1 and 2 were reduced tenfold to simulate skin where a well was present.

Interpretation

Cooper-Jacob transmissivity was estimated for each single-well test by a mechanistic approach and separately by six analysts, including the three authors and three volunteers. Transmissivity was estimated automatically with the mechanistic approach. A semilog slope was defined by the drawdowns at 0.5 and 2 d after pumping started. A minimum time of 0.5 d was selected as a compromise between avoiding the early-time complications due to wellbore storage, partial penetration, and water-table effects and assumed measurement sensitivity for actual aquifer tests. Experience guided analysts' best fit of semilog slopes. All simulated drawdowns and analyses can be retrieved from http://nevada.usgs.gov/tech/groundwater_05.htm.

Confined Analysis

Cooper-Jacob transmissivity estimates in confined aquifers were affected minimally by partial penetration, vertical anisotropy, and interpretative technique (Figure 2). Transmissivities of 100 m²/d or greater were estimated within 10 percent of their value in all but one case. A steady additional drawdown from partial penetration and vertical anisotropy was established before 12 hours of pumping had elapsed. This additional drawdown minimally affected estimates of slope and transmissivity. Confined aquifer test results were unambiguous and transmissivity estimates varied little among analysts.

Unconfined Analysis

Transmissivities of unconfined aquifers were overestimated with a mechanistic application of Cooper-Jacob (Figure 3). More than 75 percent of known transmissivity values between 10 and 1000 m²/d were overestimated. Estimates averaged twice known transmissivity values in this range. One percent of estimates were more than 10 times known transmissivity values. About 80 percent of known transmissivity values between 1000 and 10,000 m²/d were estimated within a factor of two.

Estimates by analysts were more accurate than mechanistic estimates of transmissivity (Figure 4). Analysts improved transmissivity estimates most where known transmissivity values ranged between 250 and 5000 m²/d. More than 90 percent of these transmissivity estimates were within a factor of two of the known values. Interpretation did not significantly improve transmissivity estimates or remove bias where known transmissivity values ranged between 10 and 100 m²/d.

Transmissivity estimates were not improved by interpreting results with an unconfined analytical solution instead of Cooper-Jacob. Results from an unconfined aquifer with a transmissivity of 1000 m²/d were interpreted with the Moench unconfined solution (Barlow and Moench, 1999). Transmissivity, specific storage, vertical anisotropy, specific yield, and skin were estimated simultaneously to minimize an unweighted, sum-of-squares objective function. Transmissivity estimates ranged between 800 and 1300 m²/d while matching all aspects of the drawdown curve (Figure 5). A similar transmissivity of 900 m²/d was estimated by applying Cooper-Jacob to the final drawdown limb.

Specific storage, vertical anisotropy, and specific yield could not be estimated uniquely with an unconfined Moench solution (Barlow and Moench, 1999). Estimates of these hydraulic properties with an unconfined Moench solution ranged over more than 2-orders of magnitude and several estimates were physically unreasonable (Table 2). Initial estimates strongly affected final estimates. Initial estimates were limited to plausible values for all six solutions.

Effects of Individual Parameters

Specific yield, vertical anisotropy, and partial penetration systematically affected Cooper-Jacob transmissivity estimates in unconfined aquifers (Figure 6). Effects of individual parameters on transmissivity were more pronounced on mechanistic estimates. These effects largely were eliminated by analysts where known transmissivity values ranged between 500 and 10,000 m²/d. Cooper-Jacob transmissivity estimates exceeded known transmissivity values more as specific yield increased (Figure 6a).

Decreases in vertical anisotropy proportionally displace the maximum difference between Cooper-Jacob estimates and known transmissivity (Figure 6b). Transmissivity estimates likely will be worst where vertical anisotropy ranges between 0.05 and 0.1. Estimates differ most from known transmissivity between 50 and 100 m^2/d, which is the range where errors could not be compensated with interpretation.

Partial penetration affected transmissivity estimates more than specific yield or vertical anisotropy (Figure 6c). Overestimation of transmissivity was greatest for fully penetrating wells in unconfined aquifers. Transmissivity estimates were least biased where partial penetration was 25 percent.

Empirical Corrector

An empirical corrector was developed to improve transmissivity estimates from single-well tests on unconfined aquifers. Cooper-Jacob transmissivity, vertical anisotropy, partial penetration, and specific yield define a multiplier that corrects the Cooper-Jacob transmissivity estimate. The common log of the empirical corrector (C) is

$$\log(C) = 0.63 Kxz^{0.1} (\tanh(v) + PP^{-0.5} - 2.237)\, e^{|v|(0.3-0.89Kxz^{0.1})} \quad \text{.....(1)}$$

$$v = \frac{\log(T_{CJ}) - Kxz^{0.2} - PP - \log(S_Y) - 1}{5.03\, Kxz^{0.1} 10^{\frac{|\log(4PP)|}{10}}} \quad \text{.....(2)}$$

where, Kxz is vertical anisotropy (dimensionless), PP is partial penetration (dimensionless), S_Y is specific yield (dimensionless), and T_{CJ} is a Cooper-Jacob transmissivity estimate (L^2/T). Coefficients and exponents in equations (1) and (2) were estimated by minimizing sum-of-squares differences between known transmissivity and mechanistic-transmissivity estimates. The empirical corrector is applied as a Visual Basic for Applications function in the spreadsheet *Pumping_Cooper-Jacob_wCORRECTOR.xls* and can be retrieved from http://nevada.usgs.gov/tech/groundwater_05.htm. This workbook is a modification of the previously published *Cooper-Jacob.xls* (Halford and Kuniansky, 2002).

More than 65 percent of known transmissivity values between 10 and 250 m²/d can be corrected within a factor of two (Figure 7), assuming that 95 percent of specific yield and vertical anisotropy estimates are within a factor of two of their actual values. The average of all estimates for a given transmissivity was less biased and was within 5 percent of the known value. Equation (1) is of limited value where known transmissivity exceeds 500 m²/d because limitations of the Cooper-Jacob method can be better compensated with interpretation.

Hydraulic Conductivity

Hydraulic conductivity of confined aquifers was unambiguously determined as the transmissivity estimate divided by aquifer thickness, rather than the screen length, whenever transmissivity was 100 m²/d or greater (Figure 2). Hydraulic conductivity was not defined clearly by either aquifer thickness or screen length where transmissivity was less than 50 m²/d. Screen length might be appropriate for estimating hydraulic conductivity from transmissivity but only where transmissivity is appreciably less than 10 m²/d.

Hydraulic conductivities of unconfined aquifers could not be interpreted unambiguously from transmissivity estimates with either aquifer thickness or screen length. Aquifer thickness was better than screen length for interpreting hydraulic conductivity with estimates that ranged from 0.5 to 4 times known hydraulic conductivities (Figure 6c). Hydraulic conductivity estimates ranged from 1.6 to 8 times known values where transmissivity estimates were divided by screen length.

Design Effects

Well and aquifer head losses can be better differentiated with improved transmissivity estimates from single-well aquifer tests. Head losses, from poor well completion practice (skin) and partial penetration, will be overestimated where transmissivity has been overestimated. An analyst could conclude falsely that the well was completed poorly and that completion of future wells could be improved. This scenario is likely to occur where transmissivity ranges between 10 and 250 m²/d.

Interpretive techniques from this paper can be used to improve the design of isolated production wells. Wells will more likely produce at designed rates with improved skin and transmissivity estimates because screen lengths and well diameters can be sized appropriately. The expense of poor design can be significant where deepening a well or drilling an additional well likely is a result.

Conclusions

Cooper-Jacob analysis of single well tests in unconfined aquifers overestimates transmissivity. Transmissivity estimates averaged twice known values between 10 and 1000 m^2/d and exceeded known values by more than an order of magnitude in a few cases. Transmissivity estimates also were not improved by interpreting results with an unconfined solution. Cooper-Jacob transmissivity estimates in confined aquifers differed little from known values regardless of partial penetration or vertical anisotropy. Hydraulic conductivity of confined aquifers was estimated unambiguously using the transmissivity estimate divided by aquifer thickness where transmissivity exceeded 100 m^2/d.

Transmissivity estimates from single-well tests can be improved with informed interpretation or empirical corrections. The final limb of an unconfined response can be identified by an analyst where transmissivity exceeds 500 m^2/d. All transmissivity estimates are improved qualitatively because analysts are aware of likely biases. The empirical corrector that was developed in this paper can reduce systematic bias where transmissivity ranges between 10 and 250 m^2/d. Improved transmissivity estimates should improve skin estimates that would allow for better well design. This simple approach provides a useful tool to improve transmissivity estimates from a single-well test.

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Table 1.—Well diameters and discharges assigned to simulated aquifers.

Transmissivity, m²/d	Well Diameter, m	Discharge, in m³/d
10	0.05	20
25	0.05	25
50	0.1	50
100	0.1	100
250	0.15	250
500	0.15	500
1000	0.2	1000
2500	0.3	2500
5000	0.3	5000
10,000	0.4	10,000

Table 2.—Six unconfined Moench solutions for a simulated single-well test in an aquifer with a transmissivity of 1000 m²/d, specific yield of 0.1, vertical anisotropy of 0.2, and a partial penetration of 50 percent.

Solution	Transmissivity, in m²/d	Specific Storage, in m^{-1}	Vertical Anisotropy	Specific Yield	Skin	RMS error, in m
1	360	0.000003	0.44	0.51‡	0.0	0.012
2	550	0.000005	0.35	0.33	3.0	0.008
3	790	0.000026	0.80▲	0.55‡	7.2	0.005
4	1000	0.000008	0.15	0.09	9.7	0.003
5	1300	0.0000004†	0.005▲	0.002‡	12.0	0.004
6	1400	0.000170†	1.40▲	0.52‡	17.0	0.006

†Specific-storage estimate is not in likely range of 0.000003 to 0.00003 m^{-1}.

▲Vertical anisotropy is not in likely range of 0.02 to 0.5.

‡Specific yield is not in likely range of 0.01 to 0.35.

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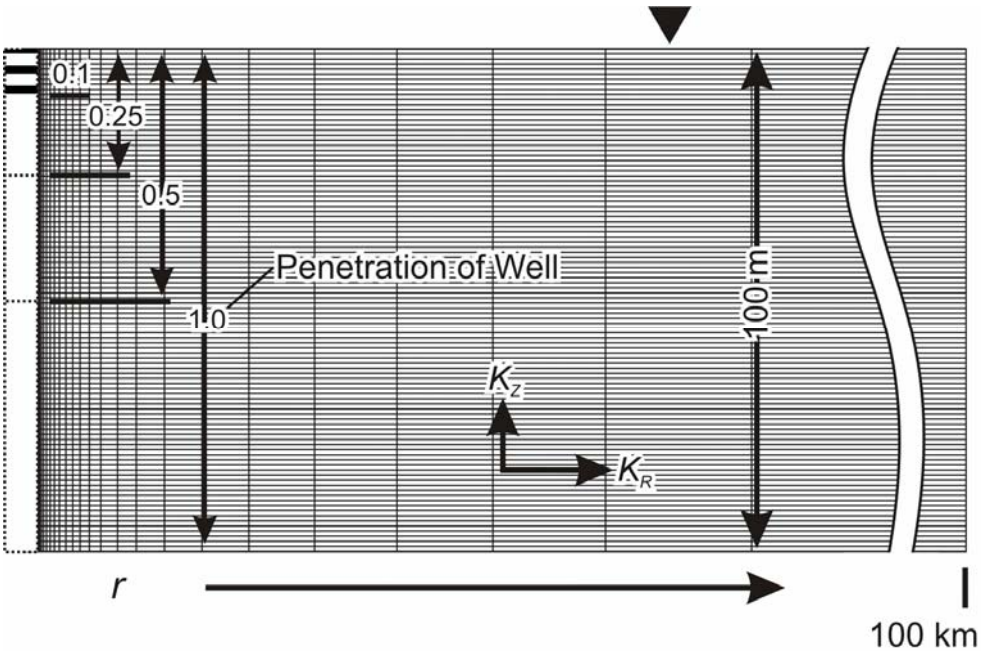

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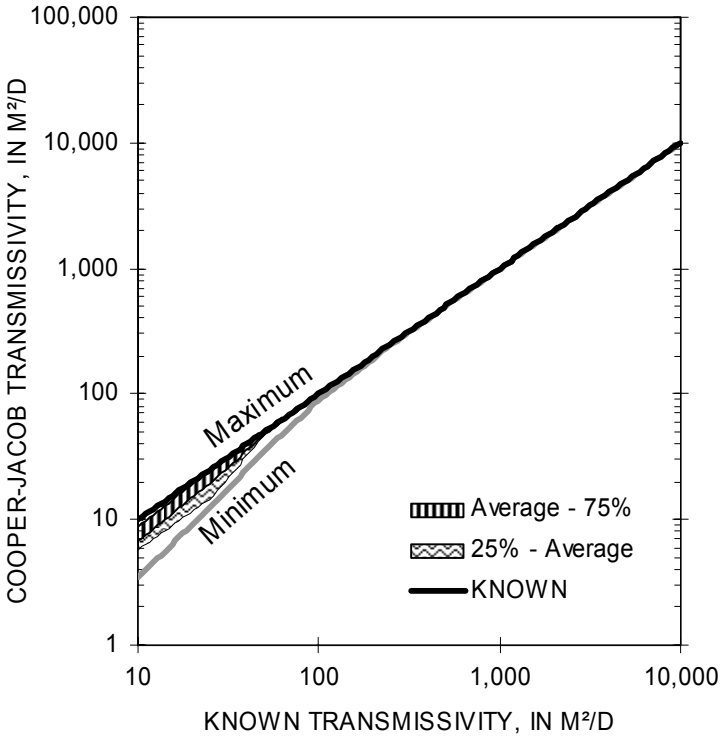

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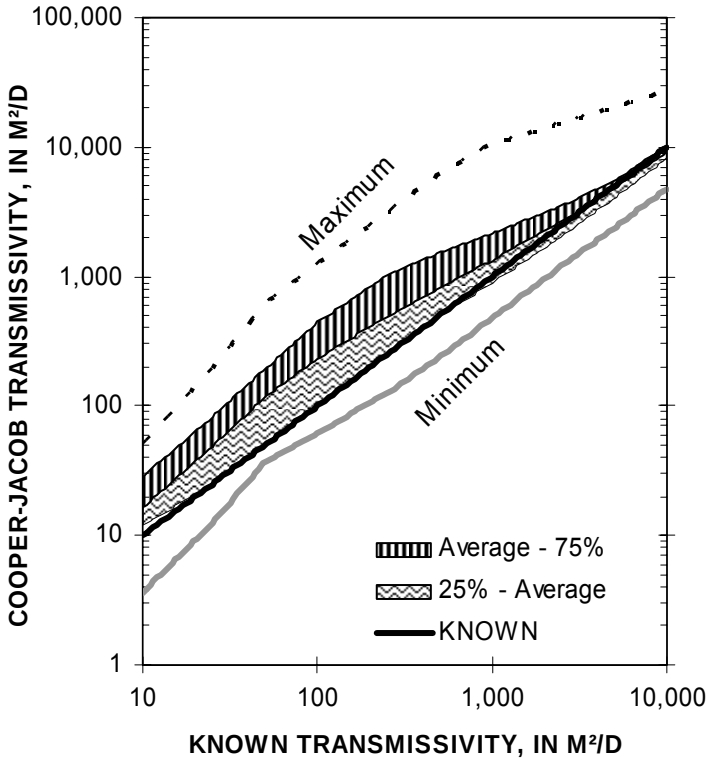

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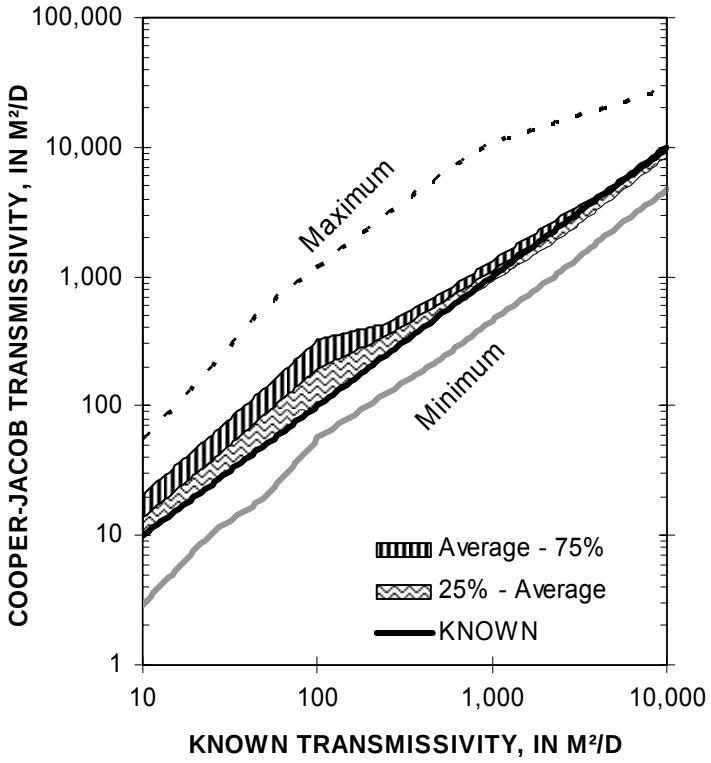

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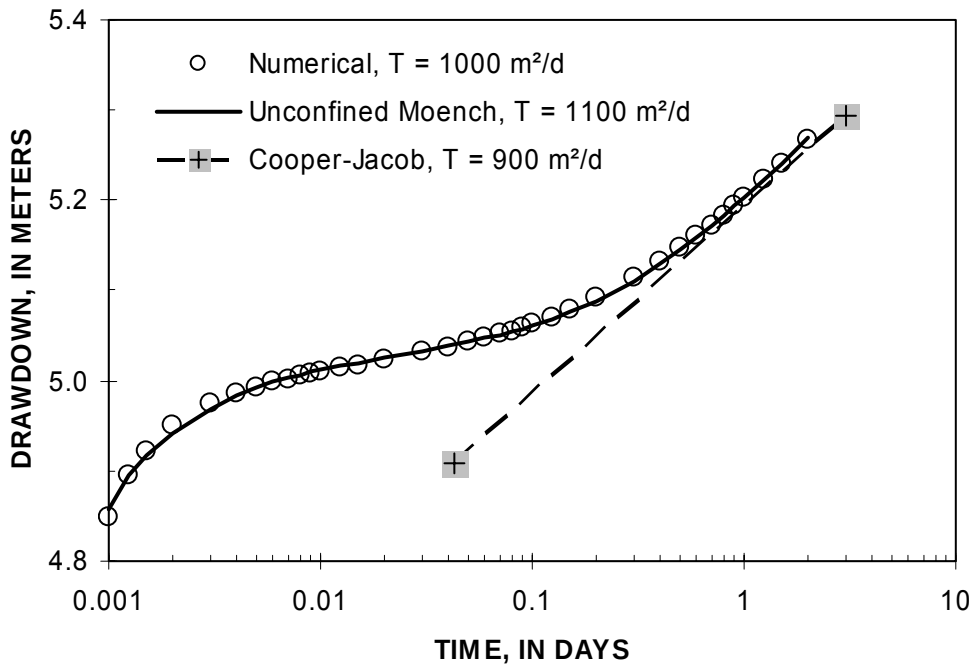

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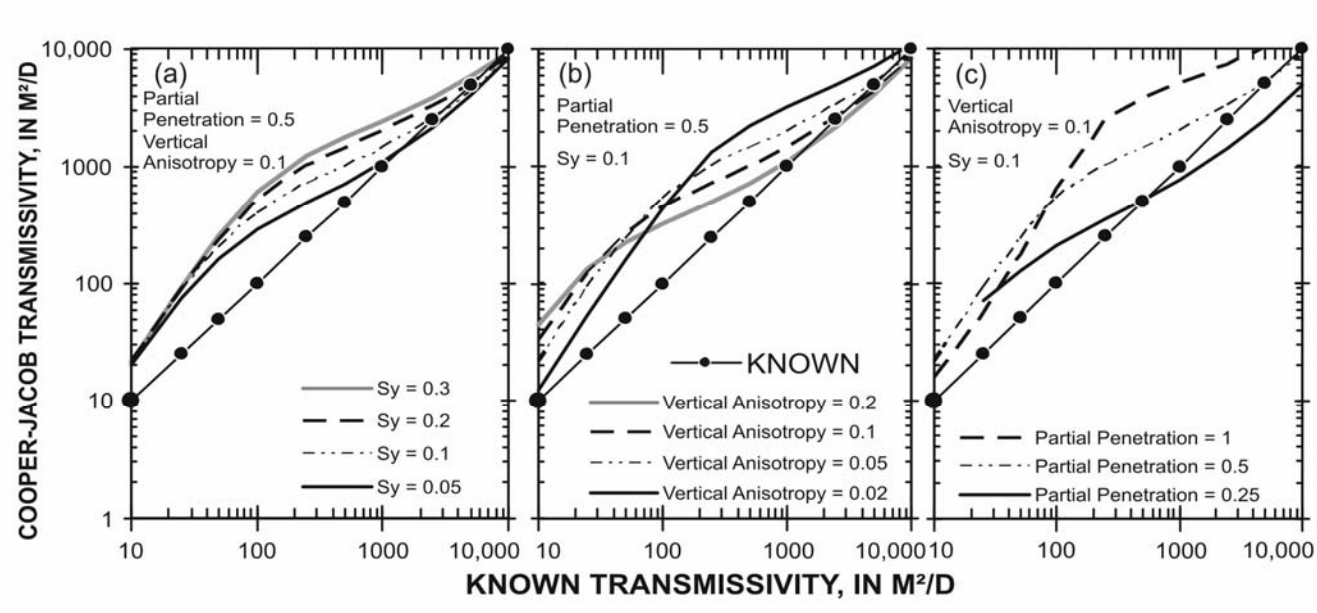

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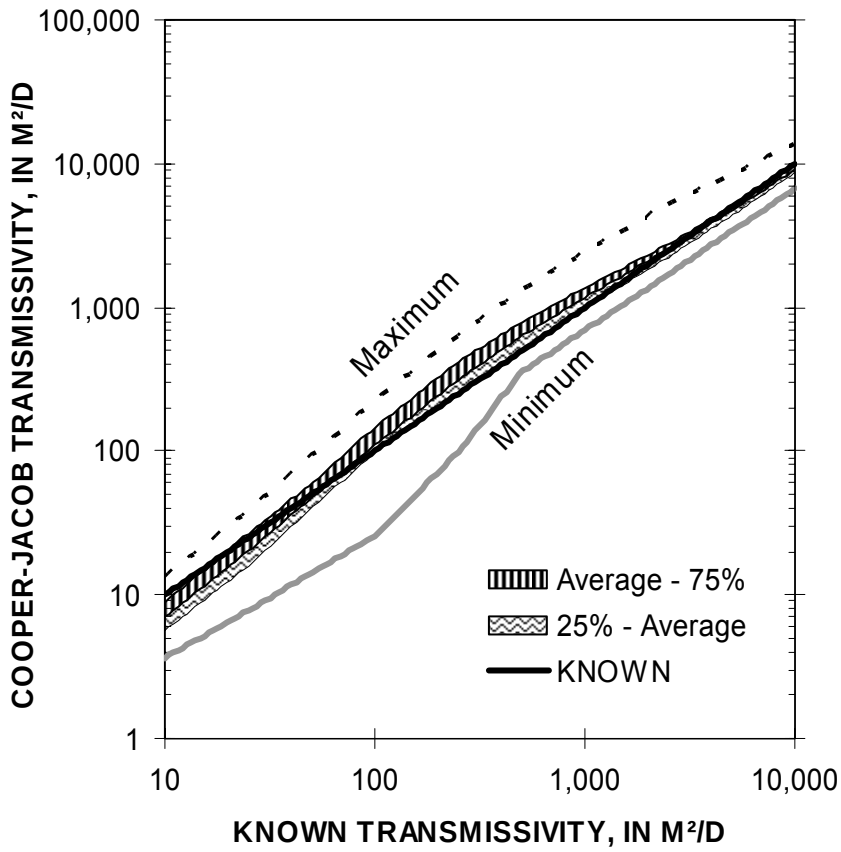

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